6.3 Notes

~~Reiman~~ Riemann Sums & Integrals

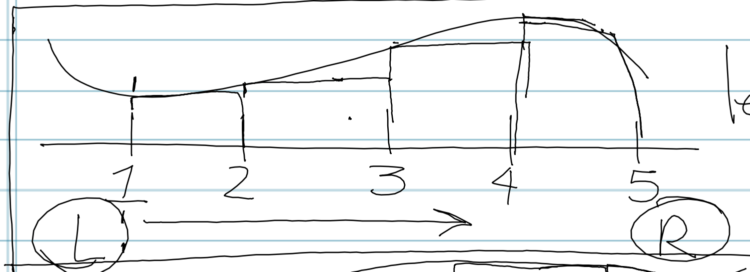
The Definite Integral: Numerical & Graphical Representations

$\sum_{i=1}^{5} i^2$ } Sumation

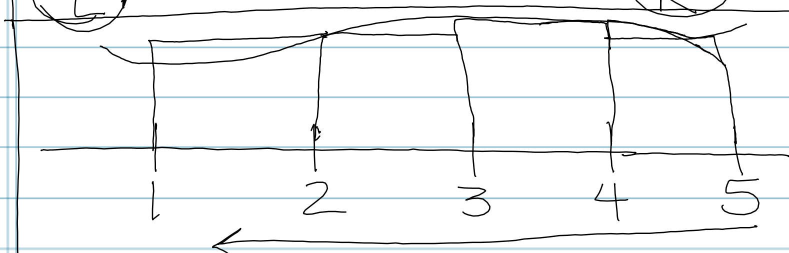
$= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 =$

$1 + 4 + 9 + 16 + 25 = \boxed{55}$

Difference between a Left & a Right Riemann Sum



left starts @ 1



Right starts @ 2

Formula to use & remember:

Left Riemann Sum: $\sum_{k=0}^{n-1} f(x_k)\Delta x$ or H * B

$f(x_k)$ = height, Δx = base

i.e. Above pts. (1–5) are $k = \emptyset$ is begin @ 1
$n-1$ is last # 5 less 1, so stop @ 4.

Right Riemann Sum [pg 482]

$$\sum_{k=1}^{n} f(x_k)\Delta x$$

Ex: #12 $f(x)=x^2$; $n=4$; $[1,5]$ L R

$$\Delta x = \frac{b-a}{n} = \frac{5-1}{4} = 1$$

Left Riemann Sum

1 2 3 4 5

$k=0$ $k=1$ $k=2$ $k=3$

now, find

$f(1) = 1^2 = 1$
$f(2) = 2^2 = 4$
$f(3) = 3^2 = 9$
$f(4) = 4^2 = 16$

Professor's way $= 30(1)$ $\boxed{=30}$

Traditional way

, Summation way is $\sum_{k=0}^{n-1} f(x_k)\Delta x = \sum_{k=0}^{3} f(x_k)\Delta x$

, So $f(1)\Delta x + f(2)\Delta x + f(3)\Delta x + f(4)\Delta x$
$= (1)^2(1) + (2)^2(1) + (3)^2(1) + (4)^2(1) = 1+4+9+16 = \boxed{30}$

If it had been a Right Riemann Sum

line

1 2 3 4 5

Professor's speedy way

$f(2) = 2^2 = 4$
$f(3) = 3^2 = 9$
$f(4) = 4^2 = 16$
$f(5) = 5^2 = 25$
$\boxed{54}$

Average Of Left/Right Riemann Sum

, so how to get the # to be more accurate estimate we take $\frac{30+54}{2} = \frac{84}{2}$
$= \boxed{42}$ better/closer average approx.

What we are actualley doing is

$$\int_1^5 x^2\,dx = \frac{x^3}{3}\Big|_1^5 = \frac{5^3}{3} - \frac{1^3}{3} = \frac{125}{3} - \frac{1}{3} = \frac{124}{3}$$

$$\boxed{= 41\tfrac{1}{3}}$$

closer to average of $\underline{\underline{42}}$.

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$$\int_a^b f(x)\,dx \cong \lim_{n\to\infty} \underbrace{\sum_{k=\emptyset}^{n-1} f(x_k)\,\Delta x}_{\text{Left Riemann Sum}}$$

Ex: $f(x) = x^3$; $n = 6$; $[1, 5]$

$$\Delta x = \frac{b-a}{n} = \frac{5-1}{6} = \frac{2}{3}$$

1 $\quad 1\tfrac{2}{3} \quad 2\tfrac{1}{3} \quad 3 \quad 3\tfrac{2}{3} \quad 4\tfrac{1}{3} \quad 5$

2/3 $\quad$ 2/3 $\quad$ 2/3 $\quad$ 2/3

Left Riemann Sum

n=6
1. $f(1) =$
2. $f(1\tfrac{2}{3}) =$
3. $f(2\tfrac{1}{3}) =$
4. $f(3) =$
5. $f(3\tfrac{2}{3}) =$
6. $f(4\tfrac{1}{3}) =$

Total $*(2/3)$ *gives approximation

1 $\quad 1\tfrac{2}{3} \quad \ldots \quad 4\tfrac{1}{3} \quad 5$

w/ Left

Right Riemann Sum

1 $\quad 1\tfrac{2}{3} \quad 4\tfrac{1}{3} \quad 5$

w/ Right

$f(1\tfrac{2}{3})$
$+ f(2\tfrac{1}{3})$
$+ f(3)$
$+ f(3\tfrac{2}{3})$
$+ f(4\tfrac{1}{3})$
$+ f(5)$

Total $* \Delta x$ = widt